Supplementary Information 1:

This Supplementary Information includes two tables summarizing the equation system solved in our numerical simulations (Tables S1–S2).

Table S1: List of the equations implemented in the CFD-DEM model

Equation names	Equations	Ref.
Mass conservation	$\frac{\partial \varepsilon_f}{\partial t} + \nabla \bullet (\varepsilon_f \vec{v}_f) = 0$	1
Momentum conservation	$\rho_f \left(\frac{\partial}{\partial t} (\varepsilon_f \vec{v}_f) + \nabla \bullet (\varepsilon_f \vec{v}_f \otimes \vec{v}_f) \right) = \nabla \bullet (\acute{\sigma}_f) + \varepsilon_{ff} \rho_f \vec{g} + \vec{I}_f$	1
Energy conservation	$(1-\phi)\rho_f C_{pf} \left(\frac{\partial T_f}{\partial t} + \vec{v}_f \bullet \nabla T_f \right) = -k_f \nabla \bullet ((1-\phi) \bullet \nabla T_f) + Q_{fs}$	1
Stress tensor	$\acute{\sigma}_f = P_f \delta_{ij} + \frac{2}{3} \eta_f tr(\acute{\epsilon}_f) \delta_{ij} + 2 \eta_f \acute{\epsilon}_f$	1
Euler velocity integration	$\vec{v}_p^{(k)}(t+\Delta t) = \vec{v}_p^{(k)}(t) + \Delta t \frac{\vec{F}_{GPD}^{(k)}(t) + \sum_{l=1}^{N_l^{(k)}} \left(\vec{F}_C^{N(k,l)}(t) + \vec{F}_C^{T(k,l)}(t) \right)}{m^{(k)}}$	Eq. (4.4)
Euler displacement integration	$\vec{X}_p^{(k)}(t+\Delta t) = \vec{X}_p^{(k)}(t) + \Delta t \vec{v}_p^{(k)}(t+\Delta t)$	2
Euler rotation integration	$\vec{\omega}_p^{(k)}(t+\Delta t) = \vec{\omega}_p^{(k)}(t) + \Delta t \frac{\sum_{l=1}^{N_l^{(k)}} \left(\vec{T}_C^{(k,l)} + \vec{T}_L^{(k,l)}(t) \right)}{I^{(k)}}$	2
Integration of solid temperature	$T_s^{(k)}(t+\Delta t) = T_s^{(k)}(t) + \Delta t_{solid} \frac{Q_{fs}^{(k)} + \sum_{l=1}^{N_l} \left(Q_{ss}^{(k,l)}(t) + Q_{sfs}^{(k,l)}(t) \right)}{m^{(k)} C_{ps}}$	2
Normal contact force	$\vec{F}_c^{N(i,j)}(t) = \left(-k_n^{(i,j)}(t) \delta_n^{(i,j)}(t) + \eta_n^{(i,j)}(t) \Delta \vec{V}_p^{N(i,j)}(t) \right) \vec{n}_{ij}$	2 5
Tangential contact force	$\vec{F}_c^{T(i,j)}(t) = -k_t^{(i,j)}(t) \delta_t^{(i,j)}(t) + \eta_t^{(i,j)}(t) \overrightarrow{\Delta V_p^{T(i,j)}}(t)$	2 5
Collisional torque	$\vec{T}_c^{(i,j)}(t) = \frac{d_p^{(i)} - \delta_n^{(i,j)}(t)}{2} \vec{F}_c^{T(i,j)}(t); \vec{T}_c^{(j,i)}(t) = \frac{d_p^{(j)} - \delta_n^{(i,j)}(t)}{2} \vec{F}_c^{T(i,j)}(t)$	2
normal spring (Hertzian model)	$k_n^{(i,j)}(t) = \frac{4}{3} \frac{E^{(i)} E^{(j)} \sqrt{R_{eff}^{(i,j)}}}{E^{(j)}(1-\sigma^{(i)2}) + E^{(i)}(1-\sigma^{(j)2})} \delta_n^{(i,j)\frac{1}{2}}(t)$	2
tangential spring (Hertzian model)	$k_t^{(i,j)}(t) = \frac{16}{3} \frac{G^{(i)} G^{(j)} \sqrt{R_{eff}^{(i,j)}}}{G^{(j)}(2-\sigma^{(i)}) + G^{(i)}(2-\sigma^{(j)})} \delta_t^{(i,j)\frac{1}{2}}(t)$	2
Elastic modulus	$G = \frac{E}{2(1+\sigma)}$	2
Normal damping coefficient	$\eta_n^{(i,j)}(t) = \frac{2\sqrt{m_{eff}^{(i,j)} k_n^{(i,j)}(t)} \lvert \ln e_n \rvert}{\sqrt{\pi^2 + \ln^2 e_n}} \delta_n^{(i,j)}(t)^{\frac{1}{4}}$	2 5
Tangential damping coefficient	$\eta_t^{(i,j)} = \frac{2\sqrt{m_{eff}^{(i,j)} k_t^{(i,j)}(t)} \lvert \ln e_t \rvert}{\sqrt{\pi^2 + \ln^2 e_t}} \delta_t^{(i,j)}(t)^{\frac{1}{4}}$	2 5

Equation names	Equations	Ref.
effective radius	$R_{eff}^{(i,j)}=\frac{2\left(d\|p^{(i)}+d_p^{(j)}\right)}{d_p^{(i)}d_p^{(j)}}$	2
Effective mass	$m_{eff}^{(i,j)}=\frac{m^{(i)}+m^{(j)}}{m^{(i)}m^{(j)}}$	2
Solids/Fluid momentum exchange on REV	$\vec{I}_f(t)=\frac{1}{\nu_{REV}}\sum_{k=1}^{N_k}\vec{F_D}^{(k)}(t)K_{REV}\left(X_p^{(k)}\right)$	2
Drag forces (for the fluid)	$\vec{F_D}^{(k)}(t)=-\nabla P_f(t)\left(\frac{\pi}{6}d_p^{(k)3}\right)+\frac{\beta_{fs}^{(k)}(t)}{\left(1-\varepsilon_f(t)\right)}\left(\frac{\pi}{6}d_p^{(k)3}\right)\left(\vec{v}_f(t)-\vec{v}_p^{(k)}(t)\right)$	2
Local fluid/solid momentum transfer	$\beta_{fs}^{(k)}(t)=\begin{cases}\frac{3}{4}C_D^{(k)}(t)\frac{\rho_f\varepsilon_f(t)(1-\varepsilon_f)\left\|\vec{v}_f-\vec{v}_s^{(k)}\right\|}{d_p^{(k)}}\varepsilon_f^{-2.65}\varepsilon_f\geq 0.8\\ \frac{150\left(1-\varepsilon_f(t)\right)^2\eta_f}{\varepsilon_f(t)d_p^{(k)2}}+\frac{1.75\rho_f\left(1-\varepsilon_f(t)\right)\left\|\vec{v}_f(t)-\vec{v}_s^{(k)}(t)\right\|}{d_p^{(k)}}\varepsilon_f<0.\end{cases}$	3 4
Drag coefficient	$C_D^{(k)}(t)=\begin{cases}\frac{24}{Re^{(k)}(t)\left(1+0.15Re^{(k)}(t)^{0.687}\right)}Re^{(k)}(t)<1000\\ 0.44Re^{(k)}(t)\geq 1000\end{cases}$	3 4
Particle Gravity-Drag-Pressure force	$\overrightarrow{F_{GPD}}(t)=\frac{m_p}{\Delta t}\left(\vec{v}_f+\tau_v\left(\vec{g}-\frac{\nabla P}{\rho_p}\right)-\vec{v}_p(t)\right)\left(1-e^{\frac{-\Delta t}{\tau_v}}\right)$	Eq. (4.5)
Reynolds number	$Re^{(k)}(t)=\frac{d_m^{(k)}\left\|\vec{v}_f(t)-\vec{v}_s^{(k)}(t)\right\|\rho_f}{\eta_f}$	3
Liquid-solid heat transfer	$Q_{fs}^{(k)}=\pi Nu^{(k)}k_f\left(T_s^{(k)}-Tf\right)$	2
Nusselt correlation	$Nu^{(k)}=2+0.6Re^{(k)\frac{1}{2}}Pr^{\frac{1}{3}}$	2
Prandle number	$Pr=\frac{\eta C_{Pf}}{k_f}$	2
Solid-Solid correlation	$Q_{SS}(i,j)=2k_sR^*(i,j)\left(T_S^{(i)}-T_S^{(j)}\right)$	2
Solid-fluid-solid conduction	$Q_{sfs}(i,j)=H(i,j)\left(T_s^{(i)}-T_s^{(j)}\right)$	2
Effective thermal conductance	$H(i,j)=\frac{H(i)H(j)}{H(i)+H(j)}$	2
Heat conductance when separated	$H^{(k)}=-k_f\int_0^{\alpha_{sfs}}\left(\frac{\pi dp^{(k)}\sin\theta}{D_{ii}^{(k,j)}-d_p^k\cos\theta}\right)d\left(\frac{dp^k}{2}\sin\theta\right)$	2
Heat conductance when in contact	$H^{(k)}=-k_f\int_{\beta_{sfs}}^{\alpha_{sfs}}\left(\frac{\pi dp^{(k)}\sin\theta}{D_{ii}^{(k,j)}-d_p^k\cos\theta}\right)d\left(\frac{dp^k}{2}\sin\theta\right)$	2

[1] Syamlal et al., (1993)
[2] Garg et al., (2010)
[3] Benyahia et al., (2012)

[4] Gidaspow, (1986)

Table S2 : Symbols used in Table S1

Symbol	Definition
$C_D^{(k)}$	Drag coefficient of the kth particle
C_{Pf}	Fluid heat capacity
$C_{Ps}^{(k)}$	Heat capacity of the kth particle
$d_p^{(i)}$	ith particle diameter
$d_p^{(i)}$	
e_n	Particle normal restitution coefficient
e_t	Particle tangential restitution coefficient
$E^{(i)}$	ith particle Young modulus
$\overrightarrow{F}_C^{N(k,l)}$	Normal contact force between kth particle and its lth neighbor
$\overrightarrow{F}_C^{T(k,l)}$	Tangential contact forces between kth particle and its lth neighbor
$\overrightarrow{F}_D^{(k)}$	Drag force on kth particle
$\vec{g}$	Gravitational vector (m s^{-2})
$G^{(k)}$	kth particle shear moduli
$h^{(i,j)}$	Distance between ith and jth particles edges
H	Heat conductance at the interface between two particles
$\vec{I}_f$	Fluid-solid momentum exchange
$I^{(k)}$	kth particle moment of inertia
K_{REM}	Generic kernel to determine the influence of a particle located at $\overrightarrow{X}_p^{(k)}$ on the REV
k_f	Fluid heat conductivity
$k_n^{(i,j)}$	Normal spring coefficient between ith and jth particles contact
$k_t^{(i,j)}$	Tangential spring coefficient between ith and jth particles contact
l	Neighbors index
$m^{(k)}$	kth particle mass
$m_{eff}^{(i,j)}$	ith and jth particles effective radius
$N_l^{(k)}$	Number of neighbors of the kth particle
N_k	Number of particles
N_u	Nusselt number
$\overrightarrow{n}_{ij}$	Normal vector between ith and jth particles
P_f	Fluid pressure (Pa)
Pr	Prandle number
Q_{fs}^k	Fluid-solid conduction of the kth particle
$Q_{ss}^{(i,j)}$	Solid-solid conduction between the ith and kth particles
$Q_{sfs}^{(i,j)}$	Solid-fluid-solid conduction between the ith and kth particles
REV	Representative elementary volume
$Re^{(k)}$	ith particle Reynolds number
$R_{eff}^{(i,j)}$	ith and jth particles effective radius
$R_{□}^{(i,j)}$	Contact area radius between ith and jth particles
R^*	Effective radius of the contact area
$\overrightarrow{T}_C^{(k,l)}$	Contact torque between kth particle and its lth neighbor
T_f	Fluid temperature
$\overrightarrow{T}_L^{(k,l)}$	Lubrication torque between kth particle and its lth neighbor
T_s^k	Temperature of the kth particle

$\vec{v}_f$	Fluid velocity vector (m s^{-1})
$\vec{v}_p^{(k)}$	kth particle velocity vector (m s^{-1})
$\vec{X}_p^{(k)}$	kth particle position (m)
$\beta_{fs}^{(k)}$	kth particle – fluid momentum transfer coefficient
$\Delta V_p^{N(i,j)}$	Normal relative velocity between ith and jth particles
$\Delta V_p^{T(i,j)}$	Tangential relative velocity between ith and jth particles
α_{sfs}	Angle between the vector relying the mass centers of the particles and the position of the edge of the thermal boundary layer
β_{sfs}	Angle between the vector relying the mass centers of the particles and the position of the edge of the contact area
δ_{ij}	Kronecker tensor
$\delta_n^{(i,j)}$	Normal overlap between ith and jth particles
$\delta_t^{(i,j)}$	Tangential displacement during the contact between ith and jth particles contact
ε	Roughness distance below which lubrication is ineffective (m)
ε_f	Fluid volume fraction
$\acute{\epsilon}_f$	Fluid strain rate tensor
η_f	Fluid viscosity (Pa s)
$\eta_n^{(i,j)}$	Normal damping coefficient between ith and jth particles
$\eta_t^{(i,j)}$	Tangential damping coefficient between ith and jth particles
θ	Incremental angle
ν	Domain volume (m^{-3})
ρ_f	Fluid density (kg m^{-3})
$\sigma^{(i)}$	ith particle Poisson coefficient
$\acute{\sigma}_f$	Fluid stress tensor
$\vec{\omega}_p^{(k)}$	kth particle rotation vector (rad s^{-1})
∇	Nabla operator
$\otimes$	Outer product